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NON–ARCHIMEDEAN GNS CONSTRUCTION AND NON–ARCHIMEDEAN KREIN—MILMAN THEOREM

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Abstract. We establish non-Archimedean analogues of the GNS construction and Krein–Milman theorem. For this purpose, we introduce notions of a state on a non-Archimedean algebra and of a convex subset of a non-Archimedean vector space. As an application, we construct two operator algebras associated to topological groups over which cyclic Banach left modules correspond to cyclic unitary representations approximated by finite dimensional cyclic semisimple unitary representations.

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