LOWER BOUNDS FOR THE NUMERICAL RADIUS

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Abstract. We show that if \( A = [a_{ij}]_{i,j=1}^n \) is an \( n \)-by- \( n \) complex matrix and \( A' = [a'_{ij}]_{i,j=1}^n \), where
\[
ad'_{ij} = \begin{cases} 
a_{ij} & \text{if } (i,j) = (1,2), \ldots, (n-1,n) \text{ or } (n,1), 
0 & \text{otherwise,}
\end{cases}
\]
then \( w(A) \geq w(A') \), where \( w(\cdot) \) denotes the numerical radius of a matrix. Moreover, if \( n \) is odd and \( a_{12}, \ldots, a_{n-1,n}, a_{n1} \) are all nonzero, then \( w(A) = w(A') \) if and only if \( A = A' \). For an even \( n \), under the same nonzero assumption, we have \( W(A) = W(A') \) if and only if \( A = A' \), where \( W(\cdot) \) is the numerical range of a matrix.

Keywords and phrases: Numerical range, numerical radius.

REFERENCES