LOWER BOUNDS FOR THE NUMERICAL RADIUS

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Abstract. We show that if $A = [a_{ij}]_{i,j=1}^n$ is an n-by-n complex matrix and $A' = [a'_{ij}]_{i,j=1}^n$, where

$$a'_{ij} = \begin{cases} a_{ij} & \text{if } (i,j) = (1,2), \dots, (n-1,n) \text{ or } (n,1), \\ 0 & \text{otherwise,} \end{cases}$$

then $w(A) \geqslant w(A')$, where $w(\cdot)$ denotes the numerical radius of a matrix. Moreover, if n is odd and $a_{12}, \dots, a_{n-1,n}, a_{n1}$ are all nonzero, then $w(A) = w(A')$ if and only if $A = A'$. For an even n, under the same nonzero assumption, we have $W(A) = W(A')$ if and only if $A = A'$, where $W(\cdot)$ is the numerical range of a matrix.

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