

ON SYMMETRY OF BIRKHOFF–JAMES ORTHOGONALITY OF LINEAR OPERATORS ON FINITE–DIMENSIONAL REAL BANACH SPACES

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Abstract. We characterize left symmetric linear operators on a finite dimensional strictly convex and smooth real normed linear space \mathbb{X} , which answers a question raised recently by one of the authors in [7] [D. Sain, *Birkhoff-James orthogonality of linear operators on finite dimensional Banach spaces*, *J. Math. Anal. Appl.* **447** (2017) 860–866]. We prove that $T \in B(\mathbb{X})$ is left symmetric if and only if T is the zero operator. If \mathbb{X} is two-dimensional then the same characterization can be obtained without the smoothness assumption. We also explore the properties of right symmetric linear operators defined on a finite dimensional real Banach space. In particular, we prove that smooth linear operators on a finite-dimensional strictly convex and smooth real Banach space can not be right symmetric.

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