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ON THE KERNEL OF A SINGULAR INTEGRAL OPERATOR WITH SHIFT

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Abstract. Some estimates for the dimension of the kernel of the singular integral operator $I - cUP_+ : L_p^n(\mathbb{T}) \to L_p^n(\mathbb{T})$, $p \in (1,\infty)$, with a non-Carleman shift are obtained, where P_+ is the Cauchy projector, U is an isometric shift operator and $c(t)$ is a continuous matrix function on the unit circle $\mathbb{T}$. It is supposed that the shift has a finite set of fixed points and all the eigenvalues of the matrix $c(t)$ at the fixed points, simultaneously belong either to the interior of the unit circle $\mathbb{T}$ or to its exterior. The case of an operator with a general shift is also considered. Some relations between those estimates and the resolvent set of the operator cU are pointed out.

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