

## INVERSE STURM–LIOUVILLE PROBLEM FOR A STAR GRAPH BY THREE SPECTRA

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*Abstract.* A three spectra problem for a star graph of three edges is solved. The given data are 1) the spectrum of a boundary value problem on the whole graph with the Dirichlet boundary conditions at the pendant vertices, continuity and Kirchhoff's conditions at the interior vertex, 2) the spectrum of the Dirichlet-Neumann problem on one of the edges, 3) the spectrum of the Dirichlet-Dirichlet problem on the union of two other edges. The aim is to find the potentials on the edges. Conditions on three sequences of numbers are found sufficient to be the spectra of these three problems.

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