

ESTIMATES ON SINGULAR VALUES OF FUNCTIONS OF PERTURBED OPERATORS

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Abstract. This is a conitunation of [1] and [2]. We prove that if function f belongs to the class $\Lambda_\omega \stackrel{\text{def}}{=} \{f : \omega_f(\delta) \leq \text{const } \omega(\delta)\}$ for an arbitrary modulus of continuity ω , then $s_j(f(A) - f(B)) \leq c \cdot \omega_* \left((1+j)^{-\frac{1}{p}} \|A - B\|_{S_p} \right) \cdot \|f\|_{\Lambda_\omega}$ for arbitrary self-adjoint operators A, B and all $1 \leq j \leq l$, where $\omega_*(x) \stackrel{\text{def}}{=} x \int_x^\infty \frac{\omega(t)}{t^2} dt$ ($x > 0$). The result is then generalized for contractions, maximal dissipative operators, normal operators and n -tuples of commuting self-adjoint operators.

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REFERENCES

- [1] A. B. ALEKSANDROV, V. V. PELLER, *Operator Hölder-Zygmund functions*, Adv. Math. **224** (2010), 910–966.
- [2] A. B. ALEKSANDROV, V. V. PELLER, *Functions of operator under perturbation of class S_p* , J. Func. Anal. **258** (2010), 3675–3724.
- [3] A. B. ALEKSANDROV, V. V. PELLER, *Functions of perturbed unbounded self-adjoint operators. Operator Bernstein type inequalities*, Indiana Univ. Math. J. **59:4** (2010), 1451–1490.
- [4] A. B. ALEKSANDROV, V. V. PELLER, *Functions of perturbed dissipative operators*, Algebra i Analiz **23** (2011), 9–51; translation in St. Petersburg Math. J. **23** (2012), 209–238.
- [5] A. B. ALEKSANDROV, V. V. PELLER, D. POTAPOV AND F. SUKOCHEV, *Functions of normal operators under perturbations*, Adv. Math. **226** (2011), 5216–5251.
- [6] A. ZYGMUND, *Trigonometric series*, 2nd ed., vols. I, II, Cambridge University Press, New York, 1959.
- [7] F. L. NAZAROV, V. V. PELLER, *Functions of n -tuples of commuting self-adjoint operators*, J. Funct. Anal. **266** (2014), 5398–5428.
- [8] M. S. BIRMAN, M. Z. SOLOMYAK, *Double Stieltjes operator integrals, III*, Problems of Math. Phys., Leningrad. Univ. **6** (1973), 27–53 (Russian).
- [9] M. S. BIRMAN, M. Z. SOLOMYAK, *Spectral theory of selfadjoint operators in Hilbert spaces*, Mathematics and its Applications (Soviet Series), D. Reidel Publishing Co., Dordrecht, 1987.
- [10] V. V. PELLER, *Hankel operators of class S_p and their applications (rational approximation, Gaussian processes, the problem of majorizing operators)*, Mat. Sbornik, **113** (1980), 538–581 (Russian), English Transl. in Math. USSR Sbornik, **41** (1982), 443–479.
- [11] V. V. PELLER, *Hankel operators in theory of perturbations of unitary and self-adjoint operators*, Funktsional. Anal. i Prilozhen. **19:2** (1985), 37–51 (Russian), English Transl. in Funct. Anal. Appl. **19** (1985), 111–123.
- [12] V. V. PELLER, *For which f does $A - B \in S_p$ imply that $f(A) - f(B) \in S_p$?*, Operator Theory, Birkhauser, **24** (1987), 289–294.
- [13] V. V. PELLER, *Hankel operators in the perturbation theory of unbounded self-adjoint operators*, Analysis and partial differential equations, 529–544, Lecture Notes in Pure and Appl. Math. **122**, Dekker, New York, 1990.