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A NOTE ON COMMUTATIVITY PRESERVING MAPS ON $M_n(\mathbb{R})$

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Abstract. Let $M_n(\mathbb{F})$ be the set of all $n \times n$ matrices over a field $\mathbb{F}$. Surjective maps which preserve the commutativity relation on $M_n(\mathbb{F})$ only in one direction have been recently classified for the case when $\mathbb{F}$ is an algebraically closed field. We show that the same result holds also when $\mathbb{F} = \mathbb{R}$ is the field of real numbers and $n \geqslant 7$ is odd.

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