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AN EXTENSION OF THE CHEN–BEURLING–HELSON–LOWDENSLAGER THEOREM

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Abstract. Yanni Chen [3] extended the classical Beurling-Helson-Lowdenslager theorem for Hardy spaces on the unit circle $\mathbb{T}$ defined in terms of continuous gauge norms on L^∞ that dominate $\|\cdot\|_1$. We extend Chen's result to a much larger class of continuous gauge norms. A key ingredient is our result that if α is a continuous normalized gauge norm on L^∞, then there is a probability measure λ, mutually absolutely continuous with respect to Lebesgue measure on $\mathbb{T}$, such that $\alpha \geqslant c\|\cdot\|_{1,\lambda}$ for some $0 < c \leqslant 1$.

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