Operators
and
Matrices
Volume 12, Number 1 (2018), 229–252
doi:10.7153/oam-2018-12-15

ON THE BLOCK NUMERICAL RANGE OF OPERATORS ON ARBITRARY BANACH SPACES

AGNES RADL AND MANFRED P. H. WOLFF

Abstract. We investigate the block numerical range of bounded linear operators on arbitrary Banach spaces. We show that the spectrum of an operator is always contained in the closure of its block numerical range. The inclusion between block numerical ranges for refined block decompositions hold only in special cases which we characterize completely. Thereby we achieve a new characterization of L^p-spaces. Finally we obtain an estimate of the resolvent in terms of the block numerical range. All our results are new even for $n \times n$-matrices.

Mathematics subject classification (2010): 47A10, 47A12, 47A75, 47B65, 15A60, 46B08, 46B42.
Keywords and phrases: Block numerical range, numerical range, spectrum, resolvent.

REFERENCES

[1] T. ANDO, *Banachverbände und positive Projektionen*, Math. Z. **109** (1969), 121–130.
[2] F. L. BAUER, J. STOER AND C. WITZGALL, *Absolute and monotonic norms*, Numer. Math. **3** (1961), 257–264.
[3] R. BHATIA, *Matrix Analysis*, Graduate Texts in Mathematics, vol. 169, Springer-Verlag, New York, 1997.
[4] F. F. BONSALL AND J. DUNCAN, *Numerical Ranges of Operators on Normed Spaces and of Elements of Normed Algebras*, London Mathematical Society Lecture Note Series **2**, Cambridge University Press, London, 1971.
[5] F. F. BONSALL AND J. DUNCAN, *Numerical Ranges, II*, London Mathematical Society Lecture Notes Series, no. 10, Cambridge University Press, New York-London, 1973.
[6] P. N. DOWLING AND B. TURETT, *Complex strict convexity of absolute norms on $\mathbb{C}^n$ and direct sums of Banach spaces*, J. Math. Anal. Appl. **323** (2006), 930–937.
[7] H. DYM, *Linear Algebra in Action*, Graduate Studies in Mathematics, vol. 78, American Mathematical Society, Providence, RI, 2007.
[8] K. H. FÖRSTER, N. HARTANTO, *On the block numerical range of nonnegative matrices. Spectral theory in inner product spaces and applications*, Oper. Theory Adv. Appl. **188**, pp. 113–133, Birkhäuser Verlag, Basel (2009).
[9] K. E. GUSTAFSON AND D. K. M. RAO, *Numerical Range. The field of values of linear operators and matrices*, Universitext, Springer-Verlag, New York, 1997.
[10] S. HEINRICH, *Ultraproducts in Banach space theory*, J. Reine Angew. Math. **313** (1980), 72–104.
[11] W. B. JOHNSON, H. B. ROSENTHAL AND M. ZIPPIN, *On bases, finite dimensional decompositions and weaker structures in Banach spaces*, Israel J. Math. **9** (1971), 488–506.
[12] P. KALLUS, *The block numerical range of operator functions in Banach spaces*, TU Berlin, Bachelor's thesis, Supervisor: K.-H. Förster, 2011.
[13] T. KATO, *Perturbation Theory for Linear Operators*, Springer, Berlin-Heidelberg-New York 1980.
[14] H. LANGER, A. MARKUS, V. MATSAEV AND C. TRETTER, *A new concept for block operator matrices: the quadratic numerical range*, Linear Algebra Appl. **330** (2001), 89–112.
[15] H. LANGER AND C. TRETTER, *Spectral decomposition of some nonselfadjoint block operator matrices*, J. Operator Theory **39** (1998), 339–359.
[16] P. MEYER-NIEBERG, *Banach lattices*, Springer, Berlin, 1991.

Paper OaM-12-15

[17] A. RADL, *Contributions to the Numerical Range and Some Applications of Semigroup Theory*, Habilitationsschrift, Leipzig, 2015.
[18] A. RADL, *The numerical range of positive operators on Banach lattices*, Positivity **19** (2015), 603–623.
[19] Y. TAKAHASHI, M. KATO AND K. S. SAITO, *Strict convexity of absolute norms on* $\mathbb{C}^2$ *and direct sums of Banach spaces*, J. Inequ. Appl. **7** (2002), 179–186.
[20] C. TRETTER AND M. WAGENHOFER, *The block numerical range of an* $n \times n$ *block operator matrix*, SIAM J. Matrix Anal. Appl. **24** (2003), 1003–1017.
[21] C. TRETTER, *Spectral Theory of Block Operator Matrices and Applications*, Imperial College Press, London, 2008.
[22] H. H. SCHAEFER, *Banach Lattices and Positive Operators*, Springer, New York, 1974.
[23] M. P. H. WOLFF, *Banach Spaces and Linear Operators*, in P. A. Loeb, M. P. H. Wolff (eds.), Nonstandard Analysis for the Working Mathematician, 2nd ed. Springer Dordrecht etc. 2015.