

A COMBINATORIAL APPROACH TO THE OPPOSITE BI-FREE PARTIAL S -TRANSFORM

PAUL SKOUFRANIS

Abstract. In this paper, we present a combinatorial approach to the opposite 2-variable bi-free partial S -transforms where the opposite multiplication is used on the right. In addition, extensions of this partial S -transforms to the conditional bi-free and operator-valued bi-free settings are discussed.

Mathematics subject classification (2010): 46L54, 46L53.

Keywords and phrases: Bi-free probability, partial bi-free S -transform, bi-free convolutions.

REFERENCES

- [1] I. CHARLESWORTH, B. NELSON, P. SKOUFRANIS, *Combinatorics of Bi-Free Probability with Amalgamation*, Comm. Math. Phys. **338**, 2 (2015), 801–847.
- [2] I. CHARLESWORTH, B. NELSON, P. SKOUFRANIS, *On two-faced families of non-commutative random variables*, Canad. J. Math. **67**, 6 (2015), 1290–1325.
- [3] K. DYKEMA, *On the S -transform over a Banach algebra*, J. Funct. Anal. **231**, 1 (2006), 90–110.
- [4] Y. GU, H.-W. HUANG, J. MINGO, *An analogue of the Lévy-Hinčin formula for bi-free infinitely divisible distributions*, to appear in Indiana Univ. Math. J., (2015), 26 pages.
- [5] Y. GU, P. SKOUFRANIS, *Conditional Bi-Free Independence for Pairs of Algebras*, arXiv:1609.07475, (2016), 44.
- [6] Y. GU, P. SKOUFRANIS, *Conditional Bi-Free Independence with Amalgamation*, arXiv:1609.07820, (2016), 36.
- [7] Y. GU, P. SKOUFRANIS, *Bi-Boolean independence for pairs of algebras*, arXiv:1703.03072, (2017), 42.
- [8] U. HAAGERUP, *On Voiculescu’s R - and S -transforms for free non-commuting variables*, “Free Probability Theory”, D. V. Voiculescu, editor, Fields Institute Communications **12** (1997), 127–148.
- [9] H.-W. HUANG, J.-C. WANG, *Analytic Aspects of Bi-Free Partial R -Transforms*, J. Funct. Anal. **271**, 4 (2016), 922–957.
- [10] H.-W. HUANG, J.-C. WANG, *Harmonic Analysis for the Bi-Free Partial S -Transform*, preprint, (2017), 28 pages.
- [11] A. NICA, R. SPEICHER, *A “Fourier Transform” for Multiplicative Functions on Non-Crossing Partitions*, J. Algebraic Combin. **6** (1997), 141–160.
- [12] M. POPA, J. C. WANG, *On multiplicative conditionally free convolution*, Trans. Amer. Math. Soc. **363**, 12 (2011), 6309–6335.
- [13] P. SKOUFRANIS, *A Combinatorial Approach to Voiculescu’s Bi-Free Partial Transforms*, Pacific J. Math. **2**, **283** (2016), 419–447.
- [14] P. SKOUFRANIS, *Independences and Partial R -Transforms in Bi-Free Probability*, Ann. Inst. Henri Poincaré Probab. Stat. **52**, 3 (2016), 1437–1473.
- [15] P. SKOUFRANIS, *On Operator-Valued Bi-Free Distributions*, Adv. Math. **303** (2016), 638–715.
- [16] R. SPEICHER, *Multiplicative functions on the lattice of non-crossing partitions and free convolution*, Math. Ann. **298**, 1 (1994), 611–628.
- [17] D. VOICULESCU, *Addition of certain non-commuting random variables*, J. Funct. Anal. **66**, 3 (1986), 323–346.

- [18] D. VOICULESCU, *Multiplication of certain non-commuting random variables*, J. Operator Theory **18** (1987), 223–235.
- [19] D. VOICULESCU, *Free probability for pairs of faces I*, Comm. Math. Phys. **332**, (2014), 955–980.
- [20] D. V. VOICULESCU, *Free Probability for Pairs of Faces II: 2-Variable Bi-Free Partial R-Transform and Systems with Rank ≤ 1 Commutation*, Ann. Inst. Henri Poincaré Probab. Stat. **52**, 1 (2016), 1–15.
- [21] D. V. VOICULESCU, *Free probability for pairs of faces III: 2-variable bi-Free partial S-transform and T-transforms*, J. Funct. Anal. **270**, 10 (2016), 3623–3638.