

## POINTWISE-GENERALIZED-INVERSES OF LINEAR MAPS BETWEEN $C^*$ -ALGEBRAS AND $JB^*$ -TRIPLES

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**Abstract.** We study pointwise-generalized-inverses of linear maps between  $C^*$ -algebras. Let  $\Phi$  and  $\Psi$  be linear maps between complex Banach algebras  $A$  and  $B$ . We say that  $\Psi$  is a pointwise-generalized-inverse of  $\Phi$  if  $\Phi(aba) = \Phi(a)\Psi(b)\Phi(a)$ , for every  $a, b \in A$ . The pair  $(\Phi, \Psi)$  is Jordan-triple multiplicative if  $\Phi$  is a pointwise-generalized-inverse of  $\Psi$  and the latter is a pointwise-generalized-inverse of  $\Phi$ . We study the basic properties of this maps in connection with Jordan homomorphism, triple homomorphisms and strongly preservers. We also determine conditions to guarantee the automatic continuity of the pointwise-generalized-inverse of continuous operator between  $C^*$ -algebras. An appropriate generalization is introduced in the setting of  $JB^*$ -triples.

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