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POINTWISE–GENERALIZED–INVERSES OF LINEAR MAPS BETWEEN C*–ALGEBRAS AND JB*–TRIPLES

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Abstract. We study pointwise-generalized-inverses of linear maps between C*-algebras. Let Φ and Ψ be linear maps between complex Banach algebras A and B. We say that Ψ is a pointwise-generalized-inverse of Φ if $\Phi(aba) = \Phi(a)\Psi(b)\Phi(a)$, for every $a,b \in A$. The pair (Φ,Ψ) is Jordan-triple multiplicative if Φ is a pointwise-generalized-inverse of Ψ and the latter is a pointwise-generalized-inverse of Φ. We study the basic properties of this maps in connection with Jordan homomorphism, triple homomorphisms and strongly preservers. We also determine conditions to guarantee the automatic continuity of the pointwise-generalized-inverse of continuous operator between C*-algebras. An appropriate generalization is introduced in the setting of JB*-triples.

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