

## DERIVATIONS ON TERNARY RINGS OF OPERATORS

ROBERT PLUTA AND BERNARD RUSSO

*Abstract.* To each projection  $p$  in a  $C^*$ -algebra  $A$  we associate a family of derivations on  $A$ , called  $p$ -derivations, and relate them to the space of triple derivations on  $pA(1-p)$ . We then show that every derivation on a ternary ring of operators is spatial and we investigate whether every such derivation on a weakly closed ternary ring of operators is inner.

*Mathematics subject classification (2010):* 47C15.

*Keywords and phrases:*  $C^*$ -algebra, ternary ring of operators, TRO, derivation, linking algebra,  $W^*$ -TRO.

### REFERENCES

- [1] T. J. BARTON, Y. FRIEDMAN, *Bounded derivations of  $JB^*$ -triples*, Quart. J. Math. Oxford Ser. (2) (1990), 255–268.
- [2] C. J. K. BATTY, *Derivations of tensor products of  $C^*$ -algebras*, J. London Math. Soc. (2) **17** (1978), 129–140.
- [3] CHO-HO CHU, *Jordan structures in geometry and analysis*, Cambridge Tracts in Mathematics, vol. 190, Cambridge University Press, Cambridge, 2012.
- [4] E. G. EFFROS, Z. J. RUAN, *Operator Spaces*, London Mathematical Society Monographs, vol. 23, Clarendon Press Oxford, 2000.
- [5] M. HAMANA, *Triple envelopes and Silov boundaries of operator spaces*, Math. J. Toyama Univ. **22** (1999), 77–93.
- [6] J. HAMHALTER, K. K. KUDAYBERGENOV, A. M. PERALTA, B. RUSSO, *Boundedness of completely additive measures with application to 2-local triple derivations*, J. Math. Physics **57** (2016), no. 2, 22 pp.
- [7] T. HO, J. MARTÍNEZ-MORENO, A. M. PERALTA, B. RUSSO, *Derivations on real and complex  $JB^*$ -triples*, J. London Math. Soc. (2) **65** (2002), no. 1, 85–102.
- [8] T. HO, A. M. PERALTA, B. RUSSO, *Ternary weakly amenable  $C^*$ -algebras and  $JB^*$ -triples*, Quarterly J. Math. **64** (2013), 1109–1139.
- [9] G. HORN, E. NEHER, *Classification of continuous  $JBW^*$ -triples*, Trans. Amer. Math. Soc. **306** (1988), 553–578.
- [10] MANMOHAN KAUR, ZHONG-JIN RUAN, *Local properties of ternary rings of operators and their linking  $C^*$ -algebras*, J. Functional Analysis **195** (2002), 262–305.
- [11] KURT MEYBERG, *Lectures on algebras and triple systems*, The University of Virginia, Charlottesville, Va., 1972.
- [12] ROBERT PLUTA, BERNARD RUSSO, *Triple derivations on von Neumann algebras*, Studia Math. **226** (2015), no. 1, 57–73.
- [13] ZHONG-JIN RUAN, *Type decomposition and the rectangular AFD property for  $W^*$ -TROs*, Canad. J. Math. **36** (2004), no. 4, 843–870.
- [14] BERNARD RUSSO, *Derivations and projections on Jordan triples: an introduction to nonassociative algebra, continuous cohomology, and quantum functional analysis*, In: Advanced courses of mathematical analysis V (2016), 118–227. World Sci. Publ., Hackensack, NJ.
- [15] S. SAKAI,  *$C^*$ -algebras and  $W^*$ -algebras*, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 60, Springer-Verlag, New York Heidelberg Berlin, 1971.
- [16] A. M. SINCLAIR, *Jordan homomorphisms and derivations on semisimple Banach algebras*, Proc. Amer. Math. Soc. **24** (1970), 209–214.

- [17] W. TIMMERMANN, *Remarks on automorphism and derivation pairs in ternary rings of unbounded operators*, Arch. Math. (Basel) **74** (2000), no. 5, 379–384.
- [18] BORUT ZALAR, *On the structure of automorphism and derivation pairs of  $B^*$ -triple systems*, Topics in operator theory, operator algebras and applications (Timisoara 1994), Rom. Acad., Bucharest (1995), 265–271.