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SHARPENING SOME CLASSICAL NUMERICAL RADIUS INEQUALITIES

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Abstract. New upper and lower bounds for the numerical radii of Hilbert space operators are given. Among our results, we prove that if $A \in \mathscr{B}(\mathscr{H})$ is a hyponormal operator, then for all non-negative non-decreasing operator convex f on $[0,\infty)$, we have

$$f(\omega(A)) \leqslant \frac{1}{2}\left\| f\left(\frac{1}{1+\frac{\xi_{|A|}^2}{8}}|A|\right) + f\left(\frac{1}{1+\frac{\xi_{|A|}^2}{8}}|A^*|\right)\right\|,$$

where $\xi_{|A|} = \inf_{\|x\|=1}\left\{\frac{\langle(|A|-|A^*|)x,x\rangle}{\langle(|A|+|A^*|)x,x\rangle}\right\}$. Our results refine and generalize earlier inequalities for hyponormal operator.

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REFERENCES

[1] A. ABU-OMAR, F. KITTANEH, *Upper and lower bounds for the numerical radius with an application to involution operators*, Rocky Mountain J. Math. **45** (2015), no. 4, 1055–1064.
[2] R. BHATIA, *Matrix Analysis*, Springer-Verlag, Berlin, 1997.
[3] S. S. DRAGOMIR, *Some inequalities for the norm and the numerical radius of linear operators in Hilbert spaces*, Tamkang J. Math. **39** (2008), no. 1, 1–7.
[4] M. FUJII, R. NAKAMOTO, *Simultaneous extensions of Selberg inequality and Heinz-Kato-Furuta inequality*, Nihonkai Math. J. **9** (1998), no. 2, 219–225.
[5] T. FURUTA, *Invitation to Linear Operators*, Taylor & Francis, Ltd., London, 2001.
[6] K. E. GUSTAFSON, D. K. M. RAO, *Numerical Range. The Field of Values of Linear Operators and Matrices*, Springer-Verlag, New York, 1997.
[7] P. R. HALMOS, *A Hilbert Space Problem Book*, 2nd ed., Springer, New York, 1982.
[8] O. HIRZALLAH, F. KITTANEH, K. SHEBRAWI, *Numerical radius inequalities for commutators of Hilbert space operators*, Numer. Funct. Anal. Optim. **32** (2011), no. 7, 739–749.
[9] O. HIRZALLAH, F. KITTANEH, K. SHEBRAWI, *Numerical radius inequalities for certain* 2×2 *operator matrices*, Integral Equations Operator Theory **71** (2011), no. 1, 129–147.
[10] L. HOGBEN (ED.), *Handbook of Linear Algebra*, Chapman & Hall/CRC Press, Boca Raton, 2007.
[11] M. KIAN, *Operator Jensen inequality for superquadratic functions*, Linear Algebra Appl. **456** (2014), 82–87.
[12] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Studia Math. **158** (2003), no. 1, 11–17.
[13] F. KITTANEH, *Norm inequalities for certain operator sums*, J. Funct. Anal. **143** (1997), no. 2, 337–348.
[14] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math. **168** (2005), no. 1, 73–80.
[15] K. SHEBRAWI, H. ALBADAWI, *Numerical radius and operator norm inequalities*, J. Inequal. Appl. (2009) Article ID 492154, 11 pages.
[16] K. SHEBRAWI, *Numerical radius inequalities for certain* 2×2 *operator matrices II*, Linear Algebra Appl. **523** (2017), 1–12.

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[17] T. YAMAZAKI, *On upper and lower bounds of the numerical radius and an equality condition*, Studia Math. **178** (2007), 83–89.
[18] L. ZOU, Y. JIANG, *Improved arithmetic-geometric mean inequality and its application*, J. Math. Inequal. **9** (2015), 107–111.