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EIGENVALUE INTERLACING FOR FIRST ORDER DIFFERENTIAL SYSTEMS WITH PERIODIC 2×2 MATRIX POTENTIALS AND QUASI–PERIODIC BOUNDARY CONDITIONS

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Abstract. The self-adjoint first order system, $JY'+QY=\lambda Y$, with locally integrable, real, symmetric, π-periodic, 2×2 matrix potential Q is considered, where $J=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. By means of a unitary transformation applied to the boundary value problem considered in [6], it is shown that all eigenvalues to the above equation with boundary conditions $Y(\pi)=\pm R(\theta)Y(0)$, where $R(\theta)$ is the rotation matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, occur when the discriminant $\Delta_\theta = Tr(\mathbb{Y}(\pi)^T R(\theta))$ is equal to ± 2. Here $\mathbb{Y}$ is the solution of the first order system obeying the initial condition $\mathbb{Y}(0)=\mathbb{I}$. In addition, an expression for the λ-derivative of the discriminant Δ_θ is given and some monotonicity results are obtained. Interlacing/indexing properties for the eigenvalues of various operator eigenvalue problems are proved.

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