EIGENVALUE INTERLACING FOR FIRST ORDER DIFFERENTIAL SYSTEMS WITH PERIODIC $2 \times 2$ MATRIX POTENTIALS AND QUASI–PERIODIC BOUNDARY CONDITIONS

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Abstract. The self-adjoint first order system, $JY' + QY = \lambda Y$, with locally integrable, real, symmetric, $\pi$–periodic, $2 \times 2$ matrix potential $Q$ is considered, where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. By means of a unitary transformation applied to the boundary value problem considered in [6], it is shown that all eigenvalues to the above equation with boundary conditions $Y(\pi) = \pm R(\theta)Y(0)$, where $R(\theta)$ is the rotation matrix $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$, occur when the discriminant $\Delta_\theta = Tr(\mathbb{Y}(\pi)^T R(\theta))$ is equal to $\pm 2$. Here $\mathbb{Y}$ is the solution of the first order system obeying the initial condition $\mathbb{Y}(0) = \mathbb{I}$. In addition, an expression for the $\lambda$–derivative of the discriminant $\Delta_\theta$ is given and some monotonicity results are obtained. Interlacing/indexing properties for the eigenvalues of various operator eigenvalue problems are proved.


Keywords and phrases: Dirac system, quasi–periodic eigenvalue problems, interlacing.

REFERENCES


