

## MAPS PRESERVING THE LOCAL SPECTRUM OF SOME MATRIX PRODUCTS

ZINE EL ABIDINE ABDELALI, ABDELALI ACHCHI AND RABI MARZOUKI

*Abstract.* Let  $\mathcal{M}_n(\mathbb{C})$  denote the algebra of all  $n \times n$  complex matrices, and  $x_0$  a nonzero vector in  $\mathbb{C}^n$ . For two fixed scalars  $\mu$  and  $\nu$  in  $\mathbb{C}$  for which  $(\mu, \nu) \neq (0, 0)$ , we characterize all maps  $\varphi$  on  $\mathcal{M}_n(\mathbb{C})$  satisfying

$$\sigma_{\mu ST^*S + \nu T^*S}(x_0) = \sigma_{\mu \varphi(S)\varphi(T)^* \varphi(S) + \nu \varphi(T)^* \varphi(S)}(x_0), \quad (S, T \in \mathcal{M}_n(\mathbb{C})).$$

This provides, in particular, a complete description of all maps on  $\mathcal{M}_n(\mathbb{C})$  preserving the local spectrum of the skew double product “ $TS^*$ ” or the skew triple product “ $TS^*T$ ” of matrices. It also unifies and extends several known results on local spectrum preservers.

*Mathematics subject classification (2010):* Primary 47B49, Secondary 47A10, 47A11.

*Keywords and phrases:* Nonlinear preservers, local spectra, skew double product, skew triple product.

### REFERENCES

- [1] Z. ABDELALI, A. ACHCHI AND R. MARZOUKI, *Maps preserving the local spectrum of skew-product of operators*, Linear Algebra Appl. **485** (2015), 58–71.
- [2] P. AIENA, *Fredholm and Local Spectral Theory, with Applications to Multipliers*, Kluwer Academic Publishers, Dordrecht, 2004.
- [3] G. AN AND J. HOU, *Rank-preserving multiplicative maps on  $\mathcal{B}(X)$* , Linear Algebra Appl. **342** (2002), 59–78.
- [4] B. AUPETIT, *A Primer on Spectral Theory*, Universitext, Springer-Verlag, New York, 1991.
- [5] M. BENDAOU, *Preservers of local spectrum of matrix Jordan triple products*, Linear Algebra Appl. **471** (2015), 604–614.
- [6] M. BENDAOU, M. JABBAR AND M. SARIH, *Preservers of local spectra of operator products*, Linear Multilinear Algebra **63** (4) (2015), 806–819.
- [7] R. BHATIA, P. ŠEMRL AND A. SOUROUR, *Maps on matrices that preserve the spectral radius distance*, Studia Math. **134** (2) (1999), 99–110.
- [8] A. BOURHIM AND M. MABROUK, *Jordan product and local spectrum preservers*, Studia Math. **234** (2) (2016), 97–120.
- [9] A. BOURHIM AND M. MABROUK, *Maps preserving the local spectrum of Jordan product of matrices*, Linear Algebra Appl. **484** (2015), 379–395.
- [10] A. BOURHIM AND J. MASHREGHI, *Local spectral radius preservers*, Integral Equations Operator Theory **76** (1) (2013), 95–04.
- [11] A. BOURHIM AND J. MASHREGHI, *Maps preserving the local spectrum of product of operators*, Glasgow Math. J. **57** (3) (2015), 709–718.
- [12] A. BOURHIM AND J. MASHREGHI, *Maps preserving the local spectrum of triple product of operators*, Linear Multilinear Algebra **63** (4) (2015), 765–773.
- [13] A. BOURHIM AND J. MASHREGHI, *A survey on preservers of spectra and local spectra*, in: Invariant Subspaces of the Shift operator, Contemp. Math. **638**, Amer. Math. Soc, Providence, RI (2015), 45–98.
- [14] A. BOURHIM AND V. G. MILLER, *Linear maps on  $\mathcal{M}_n(\mathbb{C})$  preserving the local spectral radius*, Studia Math. **188** (1) (2008), 67–75.
- [15] A. BOURHIM AND T. RANSFORD, *Additive maps preserving local spectrum*, Integral Equations Operator Theory **55** (2006), 377–385.

- [16] J. T. CHAN, C. K. LI AND N. S. SZE, *Mappings preserving spectra of products of matrices*, Proc. Amer. Math. Soc. **135** (2007), 977–986.
- [17] C. COSTARA, *Linear maps preserving operators of local spectral radius zero*, Integral Equations Operator Theory **73** (1) (2012), 7–16.
- [18] J. L. CUI AND J. C. HOU, *Maps leaving functional values of operator products invariant*, Linear Algebra Appl. **428** (2008), 1649–1663.
- [19] J. L. CUI AND C. K. LI, *Maps preserving peripheral spectrum of Jordan products of operators*, Oper. Matrices **6** (2012), 129–146.
- [20] M. DOLLINGER AND K. OBERAI, *Variation of local spectra*, J. Math. Anal. Appl. **39** (1972), 324–337.
- [21] M. GONZÁLEZ AND M. MBEKHTA, *Linear maps on  $\mathcal{M}_n(\mathbb{C})$  preserving the local spectrum*, Linear Algebra Appl. **427** (2007), 176–182.
- [22] J. C. HOU AND Q. H. DI, *Maps preserving numerical range of operator products*, Proc. Amer. Math. Soc. **134** (2006), 1435–1446.
- [23] M. MARCUS AND B. N. MOYLS, *Linear transformations on algebras of matrices*, Canad. J. Math. **11** (1959), 61–66.
- [24] T. MIURA AND D. HONMA, *A generalization of peripherally-multiplicative surjections between standard operator algebras*, Cent. Eur. J. Math. **7** (3) (2009), 479–486.
- [25] L. MOLNÁR, *Some characterizations of the automorphisms of  $B(H)$  and  $C(X)$* , Proc. Amer. Math. Soc. **130** (1) (2002), 111–120.
- [26] K. B. LAURSEN AND M. M. NEUMANN, *An Introduction to Local Spectral Theory*, London Math. Soc. Monographs (N.S.) **20**, Calderon Press, Oxford, 2000.
- [27] C. K. LI, P. ŠEMRL AND N. S. SZE, *Maps preserving the nilpotency of products of operators*, Linear Algebra Appl. **424** (2007), 222–239.
- [28] R. A. HORN AND C. R. JOHNSON, *Matrix Analysis*, 2nd edition, Cambridge University Press, Cambridge, 2012.
- [29] M. WANG, L. FANG AND G. JI, *Linear maps preserving idempotency of products or triple Jordan products of operators*, Linear Algebra Appl. **429** (2008), 181–189.
- [30] W. ZHANG AND J. HOU, *Maps preserving peripheral spectrum of Jordan semi-triple products of operators*, Linear Algebra Appl. **435** (2011), 1326–1335.