MAPS PRESERVING THE LOCAL
SPECTRUM OF SOME MATRIX PRODUCTS

ZINE EL ABIDINE ABDELALI, ABDELALI ACHCHI AND RABI MARZOUKI

Abstract. Let $\mathcal{M}_n(\mathbb{C})$ denote the algebra of all $n \times n$ complex matrices, and x_0 a nonzero vector in $\mathbb{C}^n$. For two fixed scalars μ and ν in $\mathbb{C}$ for which $(\mu, \nu) \neq (0,0)$, we characterize all maps φ on $\mathcal{M}_n(\mathbb{C})$ satisfying

$$\sigma_{\mu S T^* S + \nu T^* S}(x_0) = \sigma_{\mu \varphi(S)\varphi(T)^*\varphi(S) + \nu \varphi(T)^*\varphi(S)}(x_0), \quad (S,\ T \in \mathcal{M}_n(\mathbb{C})).$$

This provides, in particular, a complete description of all maps on $\mathcal{M}_n(\mathbb{C})$ preserving the local spectrum of the skew double product "TS^*" or the skew triple product "TS^*T" of matrices. It also unifies and extends several known results on local spectrum preservers.

Mathematics subject classification (2010): Primary 47B49, Secondary 47A10, 47A11.
Keywords and phrases: Nonlinear preservers, local spectra, skew double product, skew triple product.

REFERENCES

[1] Z. ABDELALI, A. ACHCHI AND R. MARZOUKI, *Maps preserving the local spectrum of skew-product of operators*, Linear Algebra Appl. **485** (2015), 58–71.

[2] P. AIENA, *Fredholm and Local Spectral Theory, with Applications to Multipliers*, Kluwer Academic Publishers, Dordrecht, 2004.

[3] G. AN AND J. HOU, *Rank-preserving multiplicative maps on $\mathcal{B}(X)$*, Linear Algebra Appl. **342** (2002), 59–78.

[4] B. AUPETIT, *A Primer on Spectral Theory*, Universitext, Springer-Verlag, New York, 1991.

[5] M. BENDAOUD, *Preservers of local spectrum of matrix Jordan triple products*, Linear Algebra Appl. **471** (2015), 604–614.

[6] M. BENDAOUD, M. JABBAR AND M. SARIH, *Preservers of local spectra of operator products*, Linear Multilinear Algebra **63** (4) (2015), 806–819.

[7] R. BHATIA, P. ŠEMRL AND A. SOUROUR, *Maps on matrices that preserve the spectral radius distance*, Studia Math. **134** (2) (1999), 99–110.

[8] A. BOURHIM AND M. MABROUK, *Jordan product and local spectrum preservers*, Studia Math. **234** (2) (2016), 97–120.

[9] A. BOURHIM AND M. MABROUK, *Maps preserving the local spectrum of Jordan product of matrices*, Linear Algebra Appl. **484** (2015), 379–395.

[10] A. BOURHIM AND J. MASHREGHI, *Local spectral radius preservers*, Integral Equations Operator Theory **76** (1) (2013), 95–04.

[11] A. BOURHIM AND J. MASHREGHI, *Maps preserving the local spectrum of product of operators*, Glasgow Math. J. **57** (3) (2015), 709–718.

[12] A. BOURHIM AND J. MASHREGHI, *Maps preserving the local spectrum of triple product of operators*, Linear Multilinear Algebra **63** (4) (2015), 765–773.

[13] A. BOURHIM AND J. MASHREGHI, *A survey on preservers of spectra and local spectra*, in: Invariant Subspaces of the Shift operator, Contemp. Math. **638**, Amer. Math. Soc, Providence, RI (2015), 45–98.

[14] A. BOURHIM AND V. G. MILLER, *Linear maps on $\mathcal{M}_n(\mathbb{C})$ preserving the local spectral radius*, Studia Math. **188** (1) (2008), 67–75.

[15] A. BOURHIM AND T. RANSFORD, *Additive maps preserving local spectrum*, Integral Equations Operator Theory **55** (2006), 377–385.

[16] J. T. CHAN, C. K. LI AND N. S. SZE, *Mappings preserving spectra of products of matrices*, Proc. Amer. Math. Soc. **135** (2007), 977–986.

[17] C. COSTARA, *Linear maps preserving operators of local spectral radius zero*, Integral Equations Operator Theory **73** (1) (2012), 7–16.

[18] J. L. CUI AND J. C. HOU, *Maps leaving functional values of operator products invariant*, Linear Algebra Appl. **428** (2008), 1649–1663.

[19] J. L. CUI AND C. K. LI, *Maps preserving peripheral spectrum of Jordan products of operators*, Oper. Matrices **6** (2012), 129–146.

[20] M. DOLLINGER AND K. OBERAI, *Variation of local spectra*, J. Math. Anal. Appl. **39** (1972), 324–337.

[21] M. GONZÁLEZ AND M. MBEKHTA, *Linear maps on $\mathscr{M}_n(\mathbb{C})$ preserving the local spectrum*, Linear Algebra Appl. **427** (2007), 176–182.

[22] J. C. HOU AND Q. H. DI, *Maps preserving numerical range of operator products*, Proc. Amer. Math. Soc. **134** (2006), 1435–1446.

[23] M. MARCUS AND B. N. MOYLS, *Linear transformations on algebras of matrices*, Canad. J. Math. **11** (1959), 61–66.

[24] T. MIURA AND D. HONMA, *A generalization of peripherally-multiplicative surjections between standard operator algebras*, Cent. Eur. J. Math. **7** (3) (2009), 479–486.

[25] L. MOLNÁR, *Some characterizations of the automorphisms of $B(H)$ and $C(X)$*, Proc. Amer. Math. Soc. **130** (1) (2002), 111–120.

[26] K. B. LAURSEN AND M. M. NEUMANN, *An Introduction to Local Spectral Theory*, London Math. Soc. Monographs (N.S.) **20**, Calderon Press, Oxford, 2000.

[27] C. K. LI, P. ŠEMRL AND N. S. SZE, *Maps preserving the nilpotency of products of operators*, Linear Algebra Appl. **424** (2007), 222–239.

[28] R. A. HORN AND C. R. JOHNSON, *Matrix Analysis*, 2nd edition, Cambridge University Press, Cambridge, 2012.

[29] M. WANG, L. FANG AND G. JI, *Linear maps preserving idempotency of products or triple Jordan products of operators*, Linear Algebra Appl. **429** (2008), 181–189.

[30] W. ZHANG AND J. HOU, *Maps preserving peripheral spectrum of Jordan semi-triple products of operators*, Linear Algebra Appl. **435** (2011), 1326–1335.

Operators and Matrices
www.ele-math.com
oam@ele-math.com