# NONADDITIVE STRONG COMMUTATIVITY PRESERVING MAPS ON RANK–$k$ MATRICES OVER DIVISION RINGS

CHENG-KAI LIU, PAO-KUEI LIAU AND YUAN-TSUNG TSAI

*Abstract.* Let $M_n(\mathbb{D})$ be the ring of all $n \times n$ matrices over a division ring $\mathbb{D}$, where $n \geqslant 2$ is an integer and let $\mathscr{S}$ be the set of all rank-$k$ matrices in $M_n(\mathbb{D})$, where $k$ is an integer with $1 \leqslant k \leqslant n$. We characterize maps $f : \mathscr{S} \to M_n(\mathbb{D})$ such that $[f(x), f(y)] = [x, y]$ for all $x, y \in \mathscr{S}$.

## REFERENCES

[1] A. AI, S. ALI AND V. DE FILIPPIS, *Generalized skew derivations with nilpotent values in prime rings*, Comm. Algebra **42** (2014), 1606–1618.

[2] S. ALI AND S. HUANG, *On derivations in semiprime rings*, Algebr. Represent. Theor. **15** (2012), 1023–1033.

[3] Z. BAI AND S. DU, *Strong commutativity preserving maps on rings*, Rocky Mountain J. Math. **44** (2014), 733–742.

[4] H. E. BELL AND M. N. DAIF, *On commutativity and strong commutativity preserving maps*, Canad. Math. Bull. **37** (1994), 443–447.

[5] H. E. BELL AND G. MASON, *On derivations in near rings and rings*, Math. J. Okayama Univ. **34** (1992), 135–144.

[6] M. BREŠAR, *On the distance of the composition of two derivations to the generalized derivations*, Glasgow Math. J. **33** (1991), 89–93.

[7] M. BREŠAR, *Commuting traces of biadditive mappings, commutativity preserving mappings and Lie mappings*, Trans. Amer. Math. Soc. **335** (1993), 525–546.

[8] M. BREŠAR, *Commutativity preserving maps revisited*, Israel J. Math. **162** (2007), 317–334.

[9] M. BREŠAR AND C. R. MIERS, *Strong commutativity preserving maps of semiprime rings*, Canad. Math. Bull. **37** (1994), 457–460.

[10] M. BREŠAR, M. A. CHEBOTAR AND W. S. MARTINDALE 3RD, *Functional Identities*, Birkhauser, 2007.

[11] V. DE FILIPPIS AND G. SCUDO, *Strong commutativity and Engel condition preserving maps in prime and semiprime rings*, Linear Multilinear Algebra **61** (2013), 917–938.

[12] V. DE FILIPPIS AND O. M. DI VINCENZO, *Hypercentralizing generalized skew derivations on left ideals in prime rings*, Monatsh. Math. **173** (2014), 315–341.

[13] V. DE FILIPPIS AND O. M. DI VINCENZO, *Generalized skew derivations on semiprime rings*, Linear Multilinear Algebra **63** (2015), 927–939.

[14] V. DE FILIPPIS, *Engel-type conditions involving two generalized skew derivations in prime rings*, Comm. Algebra **44** (2016), 3139–3152.

[15] Q. DENG AND M. ASHRAF, *On strong commutativity preserving maps*, Results Math. **30** (1996), 259–263.

[16] W. FRANCA, *Commuting maps on some subsets of matrices that are not closed under addition*, Linear Algebra Appl. **437** (2012), 388–391.

[17] W. FRANCA, *Commuting maps on rank-k matrices*, Linear Algebra Appl. **438** (2013), 2813–2815.

[18] W. FRANCA, *Commuting traces of multiadditive maps on invertible and singular matrices*, Linear Multilinear Algebra **61** (2013), 1528–1535.

[19] W. Franca, *Commuting traces on invertible and singular operators*, Oper. Matrices **9** (2015), 305–310.

[20] W. Franca, *Commuting traces of biadditive maps on invertible elements*, Comm. Algebra **44** (2016), 2621–2634.

[21] W. Franca, *Weakly commuting maps on the set of rank-1 matrices*, Linear Multilinear Algebra **65** (2017), 475–495.

[22] C. Hou, W. Zhang and Q. Meng, *A note on* $(\alpha, \beta)$*-derivations*, Linear Algebra Appl. **432** (2010), 2600–2607.

[23] T. W. Hungerfold, *Algebra*, Graduate Texts in Mathematics, (Book 73), Springer, 2003.

[24] T.-K. Lee and T.-L. Wong, *Nonadditive strong commutativity preserving maps*, Comm. Algebra **40** (2012), 2213–2218.

[25] C.-K. Li and S. Pierce, *Linear preserver problems*, Amer. Math. Monthly **108** (2001), 591–605.

[26] P.-K. Liau, W.-L. Huang and C.-K. Liu, *Nonlinear strong commutativity preserving maps on skew elements of prime rings with involution*, Linear Algebra Appl. **436** (2012), 3099–3108.

[27] P.-K. Liau and C.-K. Liu, *Strong commutativity preserving generalized derivations on Lie ideals*, Linear Multilinear Algebra **59** (2011), 905–915.

[28] P.-K. Liau and C.-K. Liu, *Skew derivations in Banach algebras*, Proc. Edinb. Math. Soc. **58** (2015), 683–696.

[29] J.-S. Lin and C.-K. Liu, *Strong commutativity preserving maps on Lie ideals*, Linear Algebra Appl. **428** (2008), 1601–1609.

[30] C.-K. Liu and J.-J. Yang, *Power commuting additive maps on invertible or singular matrices*, Linear Algebra Appl. **530** (2017), 127–149.

[31] C.-K. Liu, *Strong commutativity preserving generalized derivation on right ideals*, Monatsh. Math. **166** (2012), 453–465.

[32] C.-K. Liu, *The structure of triple derivations on semisimple Banach* $*$*-algebras*, Quart. J. Math. **68** (2017), 759–779.

[33] C.-K. Liu, *Centralizing maps on invertible or singular matrices over division rings*, Linear Algebra Appl. **440** (2014), 318–324.

[34] C.-K. Liu, *Strong commutativity preserving maps on some subsets of matrices that are not closed under addition*, Linear Algebra Appl. **458** (2014), 280–290.

[35] C.-K. Liu, H.-Y. Chen and P.-K. Liau, *Generalized skew derivations with nilpotent values on left ideals of rings and Banach algebras*, Linear Multilinear Algebra **62** (2014), 453–465.

[36] C.-K. Liu, *An Engel condition with skew derivations for left ideals*, Monatsh. Math. **180** (2016), 833–852.

[37] L. Liu, *Strong commutativity preserving maps on von Neumann algebras*, Linear Multilinear Algebra **63** (2015), 490–496.

[38] J. Ma, X. W. Xu and F. W. Niu, *Strong commutativity preserving generalized derivations on semiprime rings*, Acta Math. Sin. (Engl. Ser.) **24** (2008), 1835–1842.

[39] X. Qi and J. Hou, *Nonlinear strong commutativity preserving maps on prime rings*, Comm. Algebra **38** (2010), 2790–2796.

[40] X. Qi and J. Hou, *Strong commutativity preserving maps on triangular rings*, Oper. Matrices **6** (2012), 147–158.

[41] X. Qi, *Strong 2-commutativity preserving maps on prime rings*, Publ. Math. Debrecen **88** (2016), 119–129.

[42] X. Xu and H. Liu, *Additive maps on rank-$s$ matrices*, Linear Multilinear Algebra **65** (2017), 806–812.

[43] X. Xu and X. Yi, *Commuting maps on rank-k matrices*, Electron. J. Linear Algebra **27** (2014), 735–741.

[44] X. Xu, Y. Pei and X. Yi, *Additive maps on invertible matrices*, Linear Multilinear Algebra **64** (2016), 1283–1294.

[45] H. Yuan, Y. Wang, Yu. Wang and Y. Du, *Strong commutativity preserving generalized derivations on triangular rings*, Oper. Matrices **8** (2014), 773–783.