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NONADDITIVE STRONG COMMUTATIVITY PRESERVING MAPS ON RANK–k MATRICES OVER DIVISION RINGS

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Abstract. Let $M_n(\mathbb{D})$ be the ring of all $n \times n$ matrices over a division ring $\mathbb{D}$, where $n \geqslant 2$ is an integer and let $\mathscr{S}$ be the set of all rank-k matrices in $M_n(\mathbb{D})$, where k is an integer with $1 \leqslant k \leqslant n$. We characterize maps $f : \mathscr{S} \to M_n(\mathbb{D})$ such that $[f(x), f(y)] = [x, y]$ for all $x, y \in \mathscr{S}$.

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