Operators
and
Matrices
Volume 12, Number 3 (2018), 699–709
doi:10.7153/oam-2018-12-41

ON THE DENSENESS OF MINIMUM ATTAINING OPERATORS

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Abstract. Let H_1, H_2 be complex Hilbert spaces and T be a densely defined closed linear operator (not necessarily bounded). It is proved that for each $\varepsilon > 0$, there exists a bounded operator S with $\|S\| \leqslant \varepsilon$ such that $T+S$ is minimum attaining. Further, if T is bounded below, that is if there exists $m > 0$ such that $\|Tx\| \geqslant m\|x\|$ for every x in the domain of T, then S can be chosen to be rank one.

Mathematics subject classification (2010): 47A05, 47A55.
Keywords and phrases: Densely defined closed operator, minimum attaining operator, gap metric, Weyl's theorem, Lindenstrauss theorem.

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