

ON DIFFERENTIABILITY OF A CLASS OF ORTHOGONALLY INVARIANT FUNCTIONS ON SEVERAL OPERATOR VARIABLES

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Abstract. In this work, we study a connection between two classes of orthogonally invariant functions. Both types of functions are defined on $S^{n_1} \times \dots \times S^{n_k}$. The functions in the first class take their values in $S^{n_1 \dots n_k}$, while those in the second take values in $S^{\binom{n}{k}}$, where $n = n_1 + \dots + n_k$. Here, S^n denotes the set of all $n \times n$ symmetric matrices. Using that connection we establish various smoothness properties of the functions in the first class, using analogous known results about the functions in the second class.

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