

THE MODIFIED PARSEVAL EQUALITY OF STURM-LIOUVILLE PROBLEMS WITH COUPLED BOUNDARY CONDITION

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Abstract. We consider the Sturm-Liouville(S-L) problems with coupled boundary condition and transmission condition. Defining a Hilbert space related to the transmission conditions, we discuss the S-L problems in this modified Hilbert space. We prove the Parseval equality of the S-L problems with the transmission conditions in a modified Hilbert space and derive the Green's function for these problems.

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