

USING Q-CALCULUS TO STUDY LDLT FACTORIZATION OF A CERTAIN VANDERMONDE MATRIX

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Abstract. We use tools from q-calculus to study LDL^T decomposition of the Vandermonde matrix V_q with entries $v_{i,j} = q^{ij}$. We prove that the matrix L is given as a product of diagonal matrices and the lower triangular Toeplitz matrix T_q with elements $t_{i,j} = 1/(q;q)_{i-j}$, where $(z;q)_k$ is the q-Pochhammer symbol. We investigate some properties of the matrix T_q , in particular, we compute explicitly the inverse of this matrix.

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