

HYPONORMALITY OF FINITE RANK PERTURBATIONS OF NORMAL OPERATORS

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Abstract. Let T be an arbitrary finite rank perturbation of a normal operator N acting on a separable, infinite dimensional, complex Hilbert space \mathcal{H} . It is proved that the hyponormality and normality of T are equivalent. Thus every hyponormal finite rank perturbation of a normal operator has a nontrivial hyperinvariant subspace.

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