HOW TO DETERMINE THE EIGENVALUES
OF G–CIRCULANT MATRICES

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Abstract. For a given nonnegative integer g, a matrix $C_{n,g}$ of size n is called g-circulant if $C_{n,g} = [a_{(r-gs)\bmod n}]_{r,s=0}^{n-1}$. Such matrices arise in wavelet analysis, subdivision algorithms, and more generally when dealing with multigrid/multilevel methods for structured matrices and approximations of boundary value problems. In this paper, we study the eigenvalues of g-circulants. The relationship to the harmonic analysis is explored and based on the new recursive formulas for eigenvalues of such class of matrices are obtained. This result represents an extension of the work due to E. Ngondiep and S. Serra Capizzano in establishing bounds for preconditioners for the linear system of equations determined by the same matrix and it could be seen as a tool for the analysis of the preconditioners. Numerical experiments are presented to illustrate the theoretical result.

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