

ON THE WEAKLY CLOSED ALGEBRA GENERATED BY A UNITARY OPERATOR IN A PONTRYAGIN SPACE

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Abstract. This work is devoted to a study of a weakly closed algebra generated by polynomials of a unitary operator in a Pontryagin space. In particular, the inverse operator can belong or not to this algebra. The corresponding criteria is obtained.

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