

ON SUPERCYCLICITY FOR ABELIAN SEMIGROUPS OF MATRICES ON \mathbb{R}^n

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Abstract. We give a complete characterization of supercyclicity for abelian semigroups of matrices on \mathbb{R}^n , $n \geq 1$. We solve the problem of determining the minimal number of matrices over \mathbb{R} which form a supercyclic abelian semigroup on \mathbb{R}^n . In particular, we show that no abelian semigroup generated by $\left[\frac{n-1}{2}\right]$ matrices on \mathbb{R}^n can be supercyclic. ($[\cdot]$ denotes the integer part). This answers a question raised by the second author in [H. Marzougui, *Monatsh. Math.* **175** (2014), 401–410]. Furthermore, we show that supercyclicity and \mathbb{R}_+ -supercyclicity are equivalent.

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