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BESSEL PROPERTY AND BASICITY OF THE SYSTEM OF ROOT VECTOR–FUNCTIONS OF DIRAC OPERATOR WITH SUMMABLE COEFFICIENT

VALI M. KURBANOV AND AFSANA M. ABDULLAYEVA

Abstract. In the paper we study one-dimensional Dirac operator

$$Dy = By' + P(x)y, \ \ y = (y_1, \ y_2)^{\mathrm{T}},$$

where $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $P(x) = diag\,(p(x), q(x))$, $p(x)$ and $q(x)$ are complex valued functions from the class $L_1(G)$, $G = (0, 2\pi)$.

Necessary and sufficient conditions of Bessel property and unconditional basicity (the Riesz basicity) of the system of root-functions of the operator D in $L_2^2(G)$ are set up. A theorem on equivalent basicity for these systems in $L_2^2(G)$ is proved.

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