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THE SPECTRUM AND FINE SPECTRUM OF GENERALIZED RHALY–CESÀRO MATRICES ON c_0 AND c

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Abstract. The generalized Rhaly Cesàro matrices A_α are the triangular matrix with nonzero entries $a_{nk} = \alpha^{n-k}/(n+1)$ with $\alpha \in [0,1]$. In [Proc. Amer. Math. Soc. 86 (1982), 405-409], Rhaly determined boundedness, compactness of generalized Rhaly Cesàro matrices on ℓ_2 Hilbert space and shown that its spectrum is $\sigma(A_\alpha, \ell_2) = \{1/n\} \cup \{0\}$. Also in [32], lower bounds for these classes were obtained under certain restrictions on ℓ_p by Rhoades. In this paper, boundedness, compactness, spectra, the fine spectra and subdivisions of the spectra of generaled Rhaly Cesàro operator on c_0 and c have been determined.

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