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MATRIX N–DILATIONS OF QUANTUM CHANNELS

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Abstract. We study unital quantum channels which are obtained via partial trace of a $*$-automorphism of a finite unital matrix $*$-algebra. We prove that any such channel, q, on a unital matrix $*$-algebra, $\mathscr{A}$, admits a finite matrix N-dilation, α_N, for any $N \in \mathbb{N}$. Namely, α_N is a $*$-automorphism of a larger bi-partite matrix algebra $\mathscr{A} \otimes \mathscr{B}$ so that partial trace of M-fold self-compositions of α_N yield the M-fold self-compositions of the original quantum channel, for any $1 \leqslant M \leqslant N$. This demonstrates that repeated applications of the channel can be viewed as $*$-automorphic time evolution of a larger finite quantum system.

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