Operators
and
Matrices
Volume 12, Number 4 (2018), 1027–1042
doi:10.7153/oam-2018-12-62

THE EXISTENCE AND EXPRESSIONS OF THE INVERSE ALONG OPERATORS B AND C

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Abstract. For given $A,B,C \in \mathscr{B}(\mathscr{H})$, if there exists $X \in \mathscr{B}(\mathscr{H})$ such that $XAB = B$, $CAX = C$, $\mathscr{R}(X) = \mathscr{R}(B)$ and $\mathscr{R}(X^*) = \mathscr{R}(C^*)$, then A is called (B,C)-invertible and X is called the (B,C)-inverse of A. In this paper, we find some explicit properties of the one sided-inverses and (B,C)-inverses for linear bounded operators. Moreover, the solution X of the operator equations $XAB = B$ and $CAX = C$ is expressed in terms of the inner inverses of the operators A, B and C. We also present the equivalent conditions for the existence and expressions of the inverses along operators B and C.

Mathematics subject classification (2010): 15A09, 47A05.
Keywords and phrases: Block matrix, inner inverse, (B,C)-inverse.

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