

## A NOTE ON REPRESENTATIONS OF COMMUTATIVE $C^*$ -ALGEBRAS IN SEMIFINITE VON NEUMANN ALGEBRAS

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*Abstract.* In the current paper, we generalize the “compact operator” part of D. Voiculescu’s non-commutative Weyl-von Neumann theorem on approximately unitary equivalence of unital  $*$ -homomorphisms of a separable commutative  $C^*$  algebra  $\mathcal{A}$  into a semifinite von Neumann algebra. A result of D. Hadwin for approximate summands of representations into a finite von Neumann factor  $\mathcal{R}$  is also extended.

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