

ESSENTIAL NORM OF THE DIFFERENTIAL OPERATOR

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Abstract. This paper is a follow-up contribution to our work [10] where we studied some spectral properties of the differential operator D acting between generalized Fock spaces $\mathcal{F}_{(m,p)}$ and $\mathcal{F}_{(m,q)}$ when both exponents p and q are finite. In this note we continue to study the properties for the case when at least one of the spaces is growth type. We also estimate the essential norm of $D: \mathcal{F}_{(m,p)} \rightarrow \mathcal{F}_{(m,q)}$ for all $1 \leq p, q \leq \infty$, and showed that if the operator fails to be compact, then its essential norm is comparable to the operator norm and $\|D\|_e \simeq |m^{2+p} - m^{1+p}|^{\frac{1}{p}} \simeq \|D\|$.

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