

NORMWISE, MIXED AND COMPONENTWISE CONDITION NUMBERS OF MATRIX EQUATION $X - \sum_{i=1}^p A_i^T X A_i + \sum_{j=1}^q B_j^T X B_j = Q$

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Abstract. We consider a symmetric matrix equation $X - \sum_{i=1}^p A_i^T X A_i + \sum_{j=1}^q B_j^T X B_j = Q$, where $A_1, A_2, \dots, A_p, B_1, B_2, \dots, B_q \in \mathbb{R}^{n \times n}$, and Q is an $n \times n$ symmetric positive definite matrix. The explicit expressions of normwise, mixed and componentwise condition numbers of the matrix equation are investigated. Some numerical examples are given to show the sharpness of the three condition numbers.

Mathematics subject classification (2010): 15A24, 15A12, 47A05.

Keywords and phrases: Matrix equation, perturbation analysis, condition number, mixed and componentwise.

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