ORTHONORMAL SEQUENCES AND TIME FREQUENCY LOCALIZATION RELATED TO THE Riemann–Liouville OPERATOR

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Abstract. For every real number $p > 0$, we define the $p$-dispersion $\rho_{p,\nu_\alpha}(f)$ of a measurable function $f$ on $[0, +\infty) \times \mathbb{R}$, where $\nu_\alpha$ is some positive measure. We prove that for every orthonormal basis $(\phi_{m,n})_{(m,n) \in \mathbb{N}^2}$ of $L^2(d\nu_\alpha)$, the sequences $(\rho_{p,\nu_\alpha}(\phi_{m,n}))_{(m,n) \in \mathbb{N}^2}$, $(\rho_{p,\nu_\alpha}(\mathcal{F}_{\alpha}(\phi_{m,n})))_{(m,n) \in \mathbb{N}^2}$ can not be simultaneously bounded, where $\mathcal{F}_{\alpha}$ is some Fourier transform. The main tool is a time frequency localization inequality for orthonormal sequences in $L^2(d\nu_\alpha)$.

On the other hand, we construct an orthonormal sequence $(\psi_{m,n})_{(m,n) \in \mathbb{N}^2} \subset L^2(d\nu_\alpha)$ such that the sequence $(\rho_{p,\nu_\alpha}(\psi_{m,n})\rho_{p,\nu_\alpha}(\mathcal{F}_{\alpha}(\psi_{m,n})))_{(m,n) \in \mathbb{N}^2}$ is bounded.


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REFERENCES