

HYPNORMAL TOEPLITZ OPERATORS WITH NON-HARMONIC SYMBOL ACTING ON THE BERGMAN SPACE

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Abstract. The Toeplitz operator acting on the Bergman space $A^2(\mathbb{D})$, with symbol φ is given by $T_\varphi f = P(\varphi f)$, where P is the projection from $L^2(\mathbb{D})$ onto the Bergman space. We present some history on the study of hyponormal Toeplitz operators acting on $A^2(\mathbb{D})$, as well as give results for when φ is a non-harmonic polynomial. We include a first investigation of Putnam's inequality for hyponormal operators with non-analytic symbols. Particular attention is given to unusual hyponormality behavior that arises due to the extension of the class of allowed symbols. For instance, in a peculiar example, perturbation of a self-adjoint operator by a subnormal operator of arbitrarily small (though not arbitrarily large!) norm yields an operator that is not hyponormal.

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