COMPACTNESS OF OPERATOR INTEGRATORS

TITARII WOOTIJIRATTIKAL, SING-CHEONG ONG AND YONGWIMON LENBURY

Abstract. A function $f$ from a closed interval $[a, b]$ to a Banach space $X$ is a regulated function if one-sided limits of $f$ exist at every point. A function $\alpha$ from $[a, b]$ to the space $\mathcal{B}(X, Y)$, of bounded linear transformations from $X$ to a Banach space $Y$, is said to be an integrator if for each $X$-valued regulated function $f$, the Riemann-Stieltjes sums (with sampling points in the interior of subintervals) of $f$ with respect to $\alpha$ converge in $Y$. We use elementary methods to establish criteria for an integrator $\alpha$ to induce a compact linear transformation from the space, $\text{Reg}(X)$, of $X$-valued regulated functions to $Y$. We give direct and elementary proofs for each result to be used, including, among other things, the fact that each integrator $\alpha$ induces a bounded linear transformation, $\hat{\alpha}$, from $\text{Reg}(X)$ to $Y$, and other folklore or known results which required reading large amount of literature.


Keywords and phrases: Banach space, operator, regulated function, integrator, semivariation.

REFERENCES