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COMPACTNESS OF OPERATOR INTEGRATORS

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Abstract. A function f from a closed interval $[a,b]$ to a Banach space X is a *regulated function* if one-sided limits of f exist at every point. A function α from $[a,b]$ to the space $\mathfrak{B}(X,Y)$, of bounded linear transformations form X to a Banach space Y, is said to be an *integrator* if for each X-valued regulated function f, the Riemann-Stieltjes sums (with sampling points in the interior of subintervals) of f with respect to α converge in Y. We use elementary methods to establish criteria for an integrator α to induce a compact linear transformation from the space, $\text{Reg}(X)$, of X-valued regulated functions to Y. We give direct and elementary proofs for each result to be used, including, among other things, the fact that each integrator α induces a bounded linear transformation, $\widehat{\alpha}$, from $\text{Reg}(X)$ to Y, and other folklore or known results which required reading large amount of literature.

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