NONLINEAR ANTI–COMMUTING MAPS OF
STRICTLY TRIANGULAR MATRIX LIE ALGEBRAS

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Abstract. Let $N(\mathbb{F})$ be the Lie algebra consisting of all strictly upper triangular $(n+1) \times (n+1)$ matrices over a field $\mathbb{F}$. A map φ on $N(\mathbb{F})$ is called to be anti-commuting if $[\varphi(x), y] = -[x, \varphi(y)]$ for any $x, y \in N(\mathbb{F})$. We show that for $n \geqslant 4$, a nonlinear map $\varphi : N(\mathbb{F}) \to N(\mathbb{F})$ is anti-commuting if and only if there exist $b, b_1, b_2 \in \mathbb{F}$ and a nonlinear function $f : N(\mathbb{F}) \to \mathbb{F}$ such that $\varphi = ad\,(bE_{2n}) + \mu_{b_2}^{(n,n+1)} + \mu_{b_1}^{(12)} + \varphi_f$, where $ad\,(bE_{2n})$ is an inner anti-commuting map, $\mu_{b_2}^{(n,n+1)}, \mu_{b_1}^{(12)}$ are extremal anti-commuting maps, φ_f is a central anti-commuting map.

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REFERENCES

[1] D. BENKOVIČ, *Biderivations of triangular algebras*, Lin. Alge. Appl. **431** (2009): 1587–1602.

[2] D. BENKOVIČ AND D. EREMITA, *Commuting traces and commutativity preserving maps on triangular algebras*, J. Algebra **280** (2004): 797–824.

[3] J. BOUNDS, *Commuting maps over the ring of strictly upper triangular matrices*, Lin. Alge. Appl. **507** (2016): 132–136.

[4] M. BREŠAR, *Commuting maps: a survey*, Taiwanese J. Math. **8** (2004): 361–397.

[5] M. BREŠAR, *On generalized biderivations and related maps*, J. Algebra **172** (1995): 764–786.

[6] M. BREŠAR, *Commuting traces of biadditive maps, commutativity-preserving mappings and Lie mappings*, Trans. Amer. Math. Soc. **335** (1993): 525–546.

[7] M. BREŠAR AND C. R. MIERS, *Commuting maps on Lie ideals*, Comm. Algebra **23** (1995): 5539–5553.

[8] M. BREŠAR AND P. ŠEMRL, *Commuting traces of biadditive maps revisited*, Comm. Algebra **31** (2003): 381–388.

[9] L. CHEN AND J. ZHANG, *Nonlinear Lie derivations on upper triangular matrices*, Linear Multilinear Alge. 56 (2008): 725–730.

[10] Z. CHEN, *Biderivations and linear commuting maps on simple generalized Witt algebras over a field*, Electronic J. Linear Algebra **31** (2016): 1–12.

[11] Z. CHEN AND D. WANG, *Nonlinear maps satisfying derivability on standard parabolic subalgebras of finite-dimensional simple Lie algebras*, Linear Multilinear Alge. **59** (2011): 261–270.

[12] Z. CHEN AND B. WANG, *Commuting automorphisms and derivations of certain nilpotent Lie algebras over commutative rings*, Comm. Algebra **43** (2015): 2044–2061.

[13] W. CHEUNG, *Commuting maps of triangular algebras*, J. London Math. Soc. **63** (2001): 117–127.

[14] N. DIVINSKY, *On commuting automorphisms of rings*, Trans. Roy. Soc. Canada. Sect. III. **49** (1995): 19–22.

[15] G. DOLINAR, *Maps on upper triangular matrices preserving Lie products*, Linear Multilinear Alge. **55** (2007): 191–198.

[16] P. LEE, T. WONG, J. LIN AND R. WANG, *Commuting traces of multiadditive mappings*, J. Algebra **193** (1997): 709–723.

[17] T. LEE AND T. LEE, *Commuting additive mappings in semiprime rings*, Bull. Inst. Math. Acad. Sinca **24** (1996): 259–268.

[18] J. LUH, *A note on commuting automorphisms of rings*, Amer. Math. Monthly **77** (1970): 61–62.

[19] L. MOLNÁR AND P. ŠEMRL, *Non-linear commutativity preserving maps on self-adjont operators*, Q. J. Math. **56** (2005): 589–595.

[20] P. ŠEMRL, *Nonlinear commutativity preserving maps*, Acta. Sci. Math. (Szeged) **71** (2005): 781–819.

[21] X. QI AND J. HOU, *Nonlinear strong commutativity preserving maps on prime rings*, Comm. Algebra **38** (2010): 2790–2796.

[22] X. QI AND J. HOU, *Nonlinear strong commutativity preserving maps on triangular algebras*, Operators Matrices **1** (2012): 147–158.

[23] J. VUKMAN, *Commuting and centralizing mappings in prime rings*, Proc. Amer. Math. Soc. **109** (1990): 47–52.

[24] D. WANG AND X. YU, *Biderivations and linear commuting maps on the Schrödinger-Virasoro Lie algebra*, Comm. Algebra **41** (2013): 2166–2173.

[25] D. WANG, X. YU AND Z. CHEN, *Biderivations of the parabolic subalgebras of simple Lie algebras*, Comm. Algebra **39** (2011): 4097–4104.

[26] D. WANG, Y. ZHAO AND Z. CHEN, *Non-linear maps on simple Lie algebras preserving Lie products*, Comm. Algebra **39** (2011): 424–434.

[27] D. WANG, W. ZHANG AND Z. CHEN, *Product zero derivations of the parabolic subalgebras of simple Lie algebras*, J. Lie Theory **20** (2010): 167–174.

[28] J. ZHANG, S. FENG, H. LI AND R. WU, *Generalized biderivation of nest algebras*, Lin. Alge. Appl. **418** (2006): 225–233.