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SPECTRAL ESTIMATES FOR DIRICHLET LAPLACIAN ON TUBES WITH EXPLODING TWISTING VELOCITY

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Abstract. We study the spectrum of the Dirichlet Laplacian on an unbounded twisted tube with twisting velocity exploding to infinity. If the tube cross section does not intersect the axis of rotation, then its spectrum is purely discrete under some additional conditions on the twisting velocity (D. Krejčiřík, 2015). In the current work we prove a Berezin type upper bound for the eigenvalue moments.

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