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THE STIELTJES STRING AND ITS ASSOCIATED NODAL POINTS

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Abstract. Based on the theory of Stieltjes strings first introduced by Gantmakher and Krein in [4], we define the nodal points for a Stieltjes string. We show that when the eigenvalue is maximal, there are exactly $n+1$ nodal points for the D-D problem and n nodal points for the D-N problem, where n is the total number of non-zero point masses. We also find the position of these nodal points in terms of continued fractions involving the point masses $m_1,\dots,m_j$ and lengths $l_0,\dots,l_{j-1}$ in between the positions of these masses.

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