SMOOTH POINTS IN OPERATOR SPACES AND SOME
BISHOP–PHELPS–BOLLOBÁS TYPE THEOREMS IN BANACH SPACES

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Abstract. We introduce the notion of approximate norm attainment set of a bounded linear operator between Banach spaces and use it to obtain a complete characterization of smooth points in the space of compact linear operators, provided the domain space is reflexive and Kadets-Klee. We also apply the concept to characterize strong BPB property (sBPBp) of a pair of Banach spaces. We further introduce uniform $\varepsilon-$ BPB approximation of a bounded linear operator and uniform strong BPB property (uniform sBPBp) with respect to a given family of norm one linear operators and explore some of the relevant properties to illustrate its connection with earlier studies on Bishop-Phelps-Bollobás type theorems in Banach spaces. It is evident that our study has deep connections with the study of smooth points in operator spaces. We obtain a complete characterization of uniform sBPBp for a pair of Banach spaces, with respect to a given family of norm one bounded linear operators between them. As the final result of this paper, we prove that if $\mathbb{X}$ is a reflexive Kadets-Klee Banach space and $\mathbb{Y}$ is any Banach space, then the pair $(\mathbb{X}, \mathbb{Y})$ has sBPBp for compact operators. Our results extend, complement and improve some of the earlier results in this context.

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