SELF-ADJOINT OPERATORS AND THE GENERAL GKN–EM THEOREM

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Abstract. We construct self-adjoint operators in the direct sum of a complex Hilbert space $H$ and a finite dimensional complex inner product space $W$. The operator theory developed in this paper for the Hilbert space $H \oplus W$ is originally motivated by some fourth-order differential operators, studied by Everitt and others, having orthogonal polynomial eigenfunctions. Generated by a closed symmetric operator $T_0$ in $H$ with equal and finite deficiency indices and its adjoint $T_1$, we define families of minimal operators $\{\hat{T}_0\}$ and maximal operators $\{\hat{T}_1\}$ in the extended space $H \oplus W$ and establish, using a recent theory of complex symplectic geometry, developed by Everitt and Markus, a characterization of self-adjoint extensions of $\{\hat{T}_0\}$ when the dimension of the extension space $W$ is not greater than the deficiency index of $T_0$. A generalization of the classical Glazman-Krein-Naimark (GKN) theorem - called the GKN-EM theorem - is key to finding these self-adjoint extensions in $H \oplus W$. We consider several examples to illustrate our results.


Keywords and phrases: Symmetric operator, self-adjoint operator, differential operator, maximal operator, minimal operator, Glazman-Krein-Naimark theory, symplectic GKN theorem, orthogonal polynomials.

REFERENCES


