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ON THE CLASSES OF (n,m)–POWER D–NORMAL AND (n,m)–POWER D–QUASI–NORMAL OPERATORS

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Abstract. This paper is devoted to the study of some new classes of operators on Hilbert space called (n,m)-power D-normal $\left([(n,m)D\mathbf{N}]\right)$ and (n,m)-power D-quasi-normal $\left([(n,m)D\mathbf{QN}]\right)$, associated with a Drazin invertible operator using its Drazin inverse. Some properties of $[(n,m)D\mathbf{N}]$ and $[(n,m)D\mathbf{QN}]$ are investigated and some examples are also given.

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