

## UNIFORM LOCAL LIPSCHITZ CONTINUITY OF EIGENVALUES WITH RESPECT TO THE POTENTIAL IN $L^1[a, b]$

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*Abstract.* The present paper shows that the eigenvalue sequence  $\{\lambda_n(q)\}_{n \geq 1}$  of regular Sturm-Liouville eigenvalue problem with certain monotonic weights is uniformly Lipschitz continuous with respect to the potential  $q$  on any bounded subset of  $L^1([a, b], \mathbb{R})$ .

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