

APPROXIMATE EQUIVALENCE OF REPRESENTATIONS OF AF ALGEBRAS INTO SEMIFINITE VON NEUMANN ALGEBRAS

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Abstract. In this paper, we extend the “compact operator” part of Voiculescu’s theorem on approximate equivalence of unital $*$ -homomorphisms of an AF algebra when the range is in a countably decomposable, semifinite, properly infinite von Neumann factor. We also extend a result of Hadwin for approximate summands of representations into a finite von Neumann factor.

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