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MAPS PRESERVING EQUIVALENCE BY PRODUCTS OF INVOLUTIONS

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Abstract. Let $\mathscr{B}(\mathscr{X})$ be the algebra of bounded linear operators on a complex Banach space $\mathscr{X}$. Two operators A and $B \in \mathscr{B}(\mathscr{X})$ are said to be equivalent by products of involutions, if $A = TBS$ for T and S being a products of finitely many involutions. We will give description of linear bijective maps ϕ on $\mathscr{B}(\mathscr{X})$ satisfying that $\phi(A)$ and $\phi(B)$ are equivalent (i.e. $A = TBS$ for some invertible $T, S \in \mathscr{B}(\mathscr{X})$) whenever A and B are equivalent by products of involutions.

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