Operators and Matrices
Volume 13, Number 3 (2019), 867–885
doi:10.7153/oam-2019-13-62

SYLVESTER EQUATIONS AND POLYNOMIAL SEPARATION OF SPECTRA

Olavi Nevanlinna

Abstract. Sylvester equations $AX - XB = C$ have unique solutions for all C when the spectra of A and B are disjoint. Here A and B are bounded operators in Banach spaces. We discuss the existence of polynomials p such that the spectra of $p(A)$ and $p(B)$ are well separated, either inside and outside of a circle or separated into different half planes. Much of the discussion is based on the following inclusion sets for the spectrum: $V_p(T) = \{\lambda \in \mathbb{C} \,:\, |p(\lambda)| \leqslant \|p(T)\|\}$ where T is a bounded operator. We also give an explicit series expansion for the solution in terms of $p(M)$, where $M = \begin{pmatrix} A & C \\ & B \end{pmatrix}$, in the case where the spectra of A and B lie in different components of $V_p(M)$.

Mathematics subject classification (2010): 15A24, 47A10, 47A60, 47A62, 65F08, 65F10, 65J10.
Keywords and phrases: Sylvester equation, multicentric calculus, preconditioning, spectral separation.

REFERENCES

[1] R. Bhatia, P. Rosenthal, *How and Why to Solve the operator Equation AX - XB = Y*, Bull. London Math. Soc., 29 (1997) 1–21.
[2] J. W. Demmel, *The Condition Number of Equivalence Transformations that Block Diagonalize Matrix Pencils*, SIAM J. Numer. Anal. 20, No. 3, (1983).
[3] P. R.Halmos, *Capacity in Banach Algebras*, Indiana Univ. Math., 20, (1971), 855–863.
[4] Anders C. Hansen, Olavi Nevanlinna, *Complexity Issues in Computing Spectra, Pseudospectra and Resolvents*, Études Opératorielles, Banach Center Publ. Volume 112, (2017), 171–194, DOI: 10.4064/bc112-0-10.
[5] E. Heinz, *Beiträge zur Störungstheorie der Spektralzerlegung*, Math. Ann. 123 (1951) 415–438.
[6] N. J. Higham, *Functions of Matrices*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, (2008).
[7] Daniel Kressner, *Bivariate Matrix Functions*, Operators and Matrices, 8,2, (2014), 449–466, DOI : 10.7153/oam-08-23.
[8] Olavi Nevanlinna, *Convergence of Iterations for Linear Equations*, Birkhäuser, (1993).
[9] Olavi Nevanlinna, *Computing the Spectrum and Representing the Resolvent*, Numer. Funct. Anal. Optim. 30 no.9 - 10 (2009), 1025–1047.
[10] O. Nevanlinna, *Multicentric Holomorphic Calculus*, Computational Methods and Function Theory, Vol. 12, Issue 1, June 2012, 45–65.
[11] O. Nevanlinna, *Lemniscates and K -spectral sets*, J. Funct. Anal. 262, (2012), 1728–1741.
[12] O. Nevanlinna, *Polynomial as a New Variable - a Banach Algebra with Functional Calculus*, Oper. and Matrices 10 (3) (2016) 567–592.
[13] T. Ransford, *Potential Theory in the Complex Plane*, London Math. Soc. Student Texts 28, Cambridge Univ. Press, (1995).
[14] M. Rosenblum, *On the Operator Equation BX-XA=Q*, Duke Math.J. 23 (1956) 263–270.
[15] J. M. Varah, *On the Separation of Two Matrices*, SIAM J. Numer. Anal., Vol. 16(2), (1979), 216–222.