

ON THE LOCATION OF EIGENVALUES OF MATRIX POLYNOMIALS

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Abstract. A number $\lambda \in \mathbb{C}$ is called an *eigenvalue* of the matrix polynomial $P(z)$ if there exists a nonzero vector $x \in \mathbb{C}^n$ such that $P(\lambda)x = 0$. Note that each finite eigenvalue of $P(z)$ is a zero of the characteristic polynomial $\det(P(z))$. In this paper we establish some (upper and lower) bounds for eigenvalues of matrix polynomials based on the norm of their coefficient matrices and compare these bounds to those given by N. J. Higham and F. Tisseur [8], J. Maroulas and P. Psarrakos [12].

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