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THE QUINTIC COMPLEX MOMENT PROBLEM

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Abstract. Let $\gamma^{(m)} \equiv \{\gamma_{ij}\}_{0\leqslant i+j\leqslant m}$ be a given complex-valued sequence. The truncated complex moment problem (TCMP in short) involves determining necessary and sufficient conditions for the existence of a positive Borel measure μ on $\mathbb{C}$ such that $\gamma_{ij} = \int \overline{z}^i z^j d\mu$ for $0 \leqslant i+j \leqslant m$. The TCMP has been completely solved only when $m = 1, 2, 3$ and 4.

We provide in this paper a concrete solution to the, almost all, quintic TCMP (that is, when $m = 5$). We also study the cardinality of the minimal representing measure. Based on the bi-variate recurrence sequences properties with some Curto-Fialkow's results. Our method intended to be useful for all odd-degree moment problems.

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REFERENCES

[1] N. I. AKHEIZER, *The classical moment problem and some related questions in analysis*, Oliver & Boyd, 1965.
[2] C. BAYER AND J. TEICHMANN, *The proof of Tchakaloff's theorem*, Proc. Amer. Math. Soc., **134** (10) (2006), 3035–3040.
[3] G. BLEKHERMAN, *Nonnegative polynomials and sums of squares*, J. Amer. Math. Soc. **25** (3) (2012), 617–635.
[4] R. E. CURTO AND L. A. FIALKOW, *Recursively generated weighted shifts and the subnormal completion problem*, Integral Equations Operator Theory **17** (2) (1993), 202–246.
[5] R. E. CURTO AND L. A. FIALKOW, *Recursively generated weighted shifts and the subnormal completion problem II*, Integral Equations Operator Theory **18** (4) (1994), 369–426.
[6] R. E. CURTO AND L. A. FIALKOW, *Solution of the truncated complex moment problem for flat data*, Volume 568, American Mathematical Soc., 1996.
[7] R. E. CURTO AND L. A. FIALKOW, *Flat Extensions of Positive Moment Matrices: Recursively Generated Relations*, Volume 648, American Mathematical Soc., 1998.
[8] R. E. CURTO AND L. A. FIALKOW, *Flat extensions of positive moment matrices: Relations in analytic or conjugate terms*, In Nonselfadjoint operator algebras, operator theory, and related topics, pages 59–82. Springer, 1998.
[9] R. E. CURTO AND L. A. FIALKOW, *Solution of the singular quartic moment problem*, J. Operator Theory **48** (2) (2002), 315–354.
[10] R. E. CURTO AND L. A. FIALKOW, *Truncated K-moment problems in several variables*, J. Operator Theory **54** (1) (2005), 189–226.
[11] R. E. CURTO, L. A. FIALKOW, AND H. M. MÖLLER, *The extremal truncated moment problem*, Integral Equations Operator Theory, **60** (2) (2008), 177–200.
[12] R. E. CURTO AND M. PUTINAR, *Nearly subnormal operators and moment problems*, J. Funct. Anal. **115** (2) (1993), 480–497.
[13] R. E. CURTO AND S. YOO, *Non-extremal sextic moment problems*, J. Funct. Anal. **269** (3) (2015), 758–780.
[14] R. E. CURTO AND S. YOO, *Concrete solution to the nonsingular quartic binary moment problem*, Proc. Amer. Math. Soc. **144** (1) (2016), 249–258.

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[15] R. E. CURTO AND S. YOO, *The division algorithm in sextic truncated moment problems*, Integral Equations Operator Theory **87** (4) (2017), 515–528.
[16] R. E. CURTO AND S. YOO, *A new approach to the nonsingular cubic binary moment problem*, Ann. Funct. Anal. **9** (4) (2018), 525–536.
[17] O. DEMANZE, *Moments and positivity*, J. Math. Anal. Appl. **247** (2) (2000), 570–587.
[18] F. DUBEAU, W. MOTTA, M. RACHIDI, AND O. SAEKI, *On weighted r-generalized Fibonacci sequences*, The Fibonacci Quarterly **35** (2) (1997), 102–110.
[19] L. FIALKOW AND J. NIE, *Positivity of riesz functionals and solutions of quadratic and quartic moment problems*, J. Funct. Anal. **258** (1) (2010), 328–356.
[20] K. IDRISSI AND E. H. ZEROUALI, *Charges solve the truncated complex moment problem*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **21** (4) (2018), 1850027.
[21] K. IDRISSI AND E. H. ZEROUALI, *Multivariable recursively generated weighted shifts and the 2-variable subnormal completion problem*, Kyungpook Math. J., **58** (4) (2018), 711–732.
[22] D. P. KIMSEY, *Matrix-valued moment problems*, PhD thesis, Drexel University, 2011.
[23] D. P. KIMSEY, *The cubic complex moment problem*, Integral Equations Operator Theory **80** (3) (2014), 353–378.
[24] A. LAMBERT, *A characterization of subnormal operators*, Glasg. Math. J. **25** (1) (1984), 99–101.
[25] J. LASSERRE, *Polynomials nonnegative on a grid and discrete optimization*, Trans. Amer. Math. Soc., **354** (2) (2002) 631–649.
[26] J. B. LASSERRE, *Global optimization with polynomials and the problem of moments*, SIAM J. Optim. **11** (3) (2001), 796–817.
[27] J. B. LASSERRE, *Bounds on measures satisfying moment conditions*, Ann. Appl. Probab.,**12** (3) (2002), 1114–1137.
[28] J.-B. LASSERRE, *Moments, positive polynomials and their applications*, World Scientific, 2010.
[29] M. LAURENT, *Sums of squares, moment matrices and optimization over polynomials*, In Emerging applications of algebraic geometry, pages 157–270. Springer, 2009.
[30] M. PUTINAR AND C. SCHEIDERER, *Multivariate moment problems: geometry and indeterminateness*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **5** (2) (2006), 137–158.
[31] D. SARASON, *Moment problems and operators in Hilbert space, Moments in Mathematics*, Proc. Sympos. Appl. Math. **37** (1987), 54–70.
[32] Y. L. SHMUL'YAN, *An operator Hellinger integral (Russian)*, Mat. Sb. **91** (4) (1959), 381–430.
[33] J. A. SHOHAT AND J. D. TAMARKIN, *The problem of moments*, Math. Surveys I, Amer. Math. Soc., Providence, RI, 1943.
[34] J. STOCHEL, *Solving the truncated moment problem solves the full moment problem*, Glasg. Math. J. **34** (3) (2001), 335–341.
[35] V. TCHAKALOFF, *Formules de cubatures mécaniques à coefficients non négatifs*, Bull. Sci. Math. **81** (2) (1957), 123–134.
[36] F.-H. VASILESCU, *Dimensional stability in truncated moment problems*, J. Math. Anal. Appl., **388** (1) (2012), 219–230.
[37] S. YOO, *Extremal sextic truncated moment problems*, PhD thesis, The University of Iowa, 2010.
[38] S. YOO, *Sextic moment problems on 3 parallel lines*, Bull. Korean Math. Soc., **54** (1) (2017), 299–318.