$H^\infty$–FUNCTIONAL CALCULUS FOR COMMUTING FAMILIES OF RITT OPERATORS AND SECTORIAL OPERATORS

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Abstract. We introduce and investigate $H^\infty$-functional calculus for commuting finite families of Ritt operators on Banach space $X$. We show that if either $X$ is a Banach lattice or $X$ or $X^*$ has property $(\alpha)$, then a commuting $d$-tuple $(T_1,\ldots,T_d)$ of Ritt operators on $X$ has an $H^\infty$ joint functional calculus if and only if each $T_k$ admits an $H^\infty$ functional calculus. Next for $p \in (1,\infty)$, we characterize commuting $d$-tuple of Ritt operators on $L^p(\Omega)$ which admit an $H^\infty$ joint functional calculus, by a joint dilation property. We also obtain a similar characterisation for operators acting on a UMD Banach space with property $(\alpha)$. Then we study commuting $d$-tuples $(T_1,\ldots,T_d)$ of Ritt operators on Hilbert space. In particular we show that if $\|T_k\| \leq 1$ for every $k = 1,\ldots,d$, then $(T_1,\ldots,T_d)$ satisfies a multivariable analogue of von Neumann’s inequality. Further we show analogues of most of the above results for commuting finite families of sectorial operators.


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REFERENCES