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H^∞–FUNCTIONAL CALCULUS FOR COMMUTING FAMILIES OF RITT OPERATORS AND SECTORIAL OPERATORS

OLIVIER ARRIGONI AND CHRISTIAN LE MERDY

Abstract. We introduce and investigate H^∞-functional calculus for commuting finite families of Ritt operators on Banach space X. We show that if either X is a Banach lattice or X or X^* has property (α), then a commuting d-tuple $(T_1,\dots,T_d)$ of Ritt operators on X has an H^∞ joint functional calculus if and only if each T_k admits an H^∞ functional calculus. Next for $p\in(1,\infty)$, we characterize commuting d-tuple of Ritt operators on $L^p(\Omega)$ which admit an H^∞ joint functional calculus, by a joint dilation property. We also obtain a similar characterisation for operators acting on a UMD Banach space with property (α). Then we study commuting d-tuples $(T_1,\dots,T_d)$ of Ritt operators on Hilbert space. In particular we show that if $\|T_k\|\leqslant 1$ for every $k=1,\dots,d$, then $(T_1,\dots,T_d)$ satisfies a multivariable analogue of von Neumann's inequality. Further we show analogues of most of the above results for commuting finite families of sectorial operators.

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