G–DRAZIN INVERSES FOR OPERATOR MATRICES

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Abstract. Additive results for the generalized Drazin inverse of Banach space operators are presented. Suppose the bounded linear operators a and b on an arbitrary complex Banach space have generalized Drazin inverses. If $b^\pi aba = 0$ and $ab^2 = 0$, then $a+b$ has generalized Drazin inverse. This extends the main results of Djordjević and Wei (J. Austral. Math. J., **73**(2002), 115–125). Then we apply our results to 2×2 operator matrices and thereby generalize the results of Deng, Cvetković-Ilić and Wei (Linear and Multilinear Algebra, **58**(2010), 503–521).

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