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OPERATORS WITH MINIMAL PSEUDOSPECTRA AND CONNECTIONS TO NORMALITY

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Abstract. This paper mainly studies the class of bounded linear operators A with minimal pseudospectra $\sigma_\varepsilon(A) = \sigma(A) + \mathbb{D}_\varepsilon$ for some $\varepsilon > 0$, where $\sigma(A)$ denotes the spectrum of A, and $\mathbb{D}_\varepsilon$ denotes the open disk of radius ε centered at the origin. Some characterizations of the normality of operators with minimal pseudospectra are provided in terms of only one $\varepsilon-$ pseudospectrum. Furthermore, a characterization of the normality of arbitrary $N \times N$ complex matrices is given for $N \leqslant 4$. Some applications to numerical ranges are also presented.

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