

## CANONICAL FORMS OF SELF-ADJOINT BOUNDARY CONDITIONS FOR REGULAR DIFFERENTIAL OPERATORS OF ORDER THREE

TIAN NIU, XIAOLING HAO, JIONG SUN AND KUN LI

*Abstract.* In this paper, we find all canonical forms for third order self-adjoint boundary conditions. These canonical forms play an important role in the study of the dependence of the eigenvalues on the problem and for their numerical calculation. In order to obtain those canonical forms, we give a classification of self-adjoint boundary conditions. Those self-adjoint boundary conditions can be categorized into three mutually exclusive classes: coupled, strictly separated and mixed. Unlike the even order case, for the third order case, the strictly separated self-adjoint boundary conditions can not be realized. For coupled and mixed cases, there are some different types for the canonical forms: 2 for coupled and 4 for mixed boundary conditions.

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